
BREAK-EVEN POINT – QUADRATIC FUNCTIONS

While back we discussed the notions of revenue (R), cost (C), profit (P), and the break-even point:

$$\text{Profit: } P = R - C$$

$$\text{Break-even: } R = C \text{ or } P = 0$$

Using either factoring or the Quadratic Formula, we can solve more break-even business problems; these problems will result in a **quadratic equation**.



□ **BREAK-EVEN**

EXAMPLE 1: Find the break-even point(s) if the profit formula is given by $P = 2w^2 - 31w + 84$, where w is the number of widgets sold.

Solution: We find the break-even points by setting the profit formula to zero:

$$\begin{aligned}
 2w^2 - 31w + 84 &= 0 && \text{(set profit to 0)} \\
 \Rightarrow (2w - 7)(w - 12) &= 0 && \text{(factor)} \\
 \Rightarrow 2w - 7 = 0 \text{ or } w - 12 &= 0 && \text{(set each factor to 0)} \\
 \Rightarrow 2w = 7 \text{ or } w &= 12 && \text{(solve each equation)}
 \end{aligned}$$

$$\Rightarrow w = 3\frac{1}{2} \text{ or } w = 12$$

Thus, the break-even points are

$3\frac{1}{2}$ widgets and 12 widgets

Of course, $3\frac{1}{2}$ widgets can't really exist, but it's good enough for this problem. But if w stood for wages, for instance, then $3\frac{1}{2}$ would make sense, since that number represents \$3.50, a perfectly legit answer.

EXAMPLE 2: Find the break-even point(s) if revenue and cost are given by the formulas

$$R = 3w^2 - 3w - 8$$

$$C = 2w^2 + 30w - 268$$

Solution: Recall that one of the two ways to describe the **break-even points** is by equating revenue and cost. Notice that we put the resulting equation in standard form by bringing all the terms on the right side to the left side so that the right side will be zero.

$$R = C \quad \text{(to find break-even)}$$

$$\Rightarrow 3w^2 - 3w - 8 = 2w^2 + 30w - 268 \quad \text{(use the given formulas)}$$

$$\Rightarrow w^2 - 3w - 8 = 30w - 268 \quad \text{(subtract } 2w^2 \text{)}$$

$$\Rightarrow w^2 - 33w - 8 = -268 \quad \text{(subtract } 30w \text{)}$$

$$\Rightarrow w^2 - 33w + 260 = 0 \quad \text{(add 268 } \Rightarrow \text{ standard form)}$$

$$\Rightarrow (w - 20)(w - 13) = 0 \quad \text{(factor)}$$

$$\Rightarrow w - 20 = 0 \text{ or } w - 13 = 0 \quad \text{(set each factor to 0)}$$

$$\Rightarrow w = 20 \text{ or } w = 13 \quad \text{(solve each equation)}$$

And so the two break-even points are

20 widgets and 13 widgets

Homework

1. Find the **break-even points** for the given profit formula:
 - a. $P = w^2 - 12w + 35$ b. $P = 2w^2 - 13w + 15$
 - c. $P = w^2 - 25w + 150$ d. $P = 6w^2 - 31w + 40$

2. Find the **break-even points** for the given revenue and cost formulas:
 - a. $R = 2w^2 + 8w - 20$ $C = w^2 + 24w - 75$
 - b. $R = 2w^2 - 3w + 1$ $C = -w^2 + 16w - 29$
 - c. $R = 5w^2 - 26w + 80$ $C = 3w^2 + w + 10$

Review Problems

3. If the profit formula is given by $P = w^2 - 57w + 350$, find the two break-even points.

4. If the revenue and cost are given by the formulas $R = 5w^2 - 30w + 100$ and $C = 4w^2 + 4w - 180$, find the break-even points.

Solutions

1. a. $w = 5, 7$ b. $w = \frac{3}{2}, 5$ c. $w = 10, 15$ d. $w = \frac{8}{3}, \frac{5}{2}$
2. a. $w = 5, 11$ b. $w = \frac{10}{3}, 3$ c. $w = \frac{7}{2}, 10$
3. $w = 7$ and $w = 50$ 4. $w = 14$ and $w = 20$

“There is one purpose to life and one only: to bear witness to and understand as much as possible of the complexity of the world – its beauty, its mysteries, its riddles. The more you

understand, the more you look – the greater is your enjoyment of life and your sense of peace. That's all there is to it. If an activity is not grounded in ‘*to love*’ or ‘*to learn*,’ it does not have value.”

– Anne Rice, American Author